Waveguide Bandstop Elliptic Function Filters

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Abstract—**Using the "natural prototype" for elliptic function filters, a design procedure is presented for a class of waveguide bandstop filters, which exhibit equiripple passband and stopband responses. Due to the availability of explicit formulas for element values in the natural prototype elliptic function filter, the design procedure is entirely analytic and does not require numerical synthesis techniques.**

The resulting physical structure is the familiar uniform guide with iris-coupled series stubs. Unlike the bandstop filters designed from maximally flat or Chebyshev prototypes, the elliptic function design results in stubs that are not exactly three-quarter-wave coupled.

Introduction

Recently, design procedures have become available for waveguide bandpass elliptic function filters [1]–[3]. The latter is based upon the conventional prototype shown in Fig. 1(a) and the former two on similar modified versions. If this prototype were to be used to produce waveguide bandstop filters, dual mode cavities would normally be required to realize pairs of transmission zeros in the bandstop region and interaction between modes would become difficult to control.

In a recent paper [4], a new prototype for elliptic function filters has been devised with the added advantage of explicit formulas for element values. In this "natural prototype," shown in Fig. 1(b) for the high-pass case, the transmission zeros are realized individually in a direct ladder form, thus requiring only single cavities in any bandstop region and allowing the normal configuration [5] for waveguide bandstop filters to be used. This is shown in Fig. 2(a) where the iris-coupled series stubs are normally separated by approximately three-quarters of a wavelength to avoid evanescent mode interaction [5]. Explicit design equations are presented for the waveguide bandstop elliptic function filter realized in this form.

Element Values for High-Pass Prototype Elliptic Function Filters [4]

For the nth degree network shown in Fig. 1(b), which exhibits the optimum elliptic function insertion loss, as shown in Fig. 2(b), of the form

$$L = 10 \log \left[1 + \frac{1}{\epsilon^2 \left[\mathrm{cd}_0 \left(\frac{nK_0}{K} \mathrm{cd}^{-1} \omega \right) \right]^2} \right] \tag{1}$$

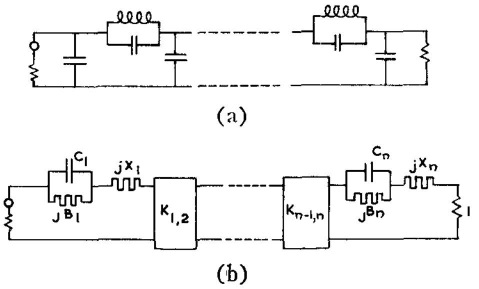

Fig. 1. (a) Conventional low-pass prototype elliptic function filter. (b) Natural prototype elliptic function filter.

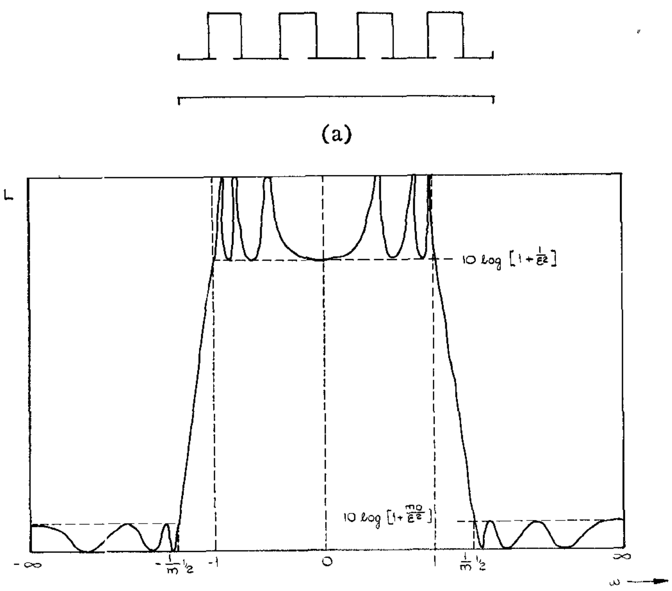

Fig. 2. (a) Symmetrical bandstop filter. (b) Insertion loss function for high-pass prototype.

where the subscript "0" indicates dependence on the elliptic parameter m_0, otherwise on the parameter m, the element values are, for $r = 1 \rightarrow n$,

$$C_r = \frac{\mathrm{ds}\left[\frac{(2r-1)K}{n}\right] \mathrm{dn}\left[\frac{(2r-1)K}{n}\right]}{2y(1-m)} \tag{2}$$

$$B_r = C_r \, \mathrm{cd}\left[\frac{(2r-1)K}{n}\right] \tag{3}$$

$$R_r = -\gamma m \left[\mathrm{sn}\left[\frac{2(r-1)K}{n}\right] + \mathrm{sn}\left[\frac{2rK}{n}\right] \cdot \mathrm{cd}\left[\frac{K}{n}\right] \mathrm{cd}\left[\frac{(2r-1)K}{n}\right]\right] \tag{4}$$

Manuscript received December 28, 1970; revised June 12, 1972.

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and, for $r = 1 \to n-1$,

$$K_{r,r+1} = \sqrt{1 + y^2 m\, \mathrm{sn}^2\left[\frac{2rK}{n}\right]} \tag{5}$$

where

$$\mathrm{sc}\left[\frac{nK_0U}{K}\middle|\, 1 - m_0\right] = \frac{1}{\epsilon} \qquad y = \mathrm{sc}\,[U \mid 1 - m] \tag{6}$$

with the conditional requirement

$$\frac{nK_0}{K} = \frac{K_0'}{K'}. \tag{7}$$

In (1)–(5) and (7), the Jacobian elliptic functions are dependent on the parameter m.

The independent parameters in these formulas are ϵ, n, and m. If ω' is the ratio of the required passband bandwidth to the stopband bandwidth we have

$$\omega' = \frac{1}{m^{1/2}} \tag{8}$$

and the parameters ϵ and n may be directly related to L_S and L_R, the transmission loss in the stopband and the return loss in the passband, respectively, in the following manner:

$$L_S = 10 \log\left[1 + \frac{1}{\epsilon^2}\right]$$
$$L_R = 10 \log\left[1 + \frac{\epsilon^2}{m_0}\right] \tag{9}$$

or

$$L_S \approx 20 \log\left[\frac{1}{\epsilon}\right]$$
$$L_R \approx 20 \log\left[\frac{\epsilon}{m_0^{1/2}}\right] \tag{10}$$

resulting in

$$L_S + L_R = 10 \log\left[\frac{1}{m_0}\right]. \tag{11}$$

For m_0 small,

$$m_0 \approx 16 e^{-\pi K_0'/K_0}. \tag{12}$$

From (11) and (12) we therefore have the basic design equation

$$13.65 \frac{nK'}{K} - 12 = L_R + L_S. \tag{13}$$

As an example, let $L_R + L_S = 45$ dB and $\omega' = 1/m^{1/2} = 1.25((K'/K) = 0.88)$. From (13),

$$n \geq \frac{45 + 12}{13.65 \times 0.88} = 4.8 \tag{14}$$

i.e., $n = 5$.

Comparison with standard tables on Chebyshev filters will show that a nine-cavity filter is required to meet the above specification, illustrating the superiority of the elliptic function response.

Symmetrical Bandstop Filter (Odd Degree)

To convert the basic natural prototype shown in Fig. 1(b) into a form that is readily realizable in waveguide, it is first necessary to eliminate the frequency invariant reactances in the prototype by using phase shifters of unity characteristic impedance defined by a transfer matrix of the form

$$\begin{bmatrix} \cos\theta & j\sin\theta \\ j\sin\theta & \cos\theta \end{bmatrix}. \tag{15}$$

Consider a basic series element in the prototype network as shown in Fig. 3(a) where

$$Y_r = [\omega C_r + B_r]$$
$$Z_r = R_r. \tag{16}$$

This element is then decomposed as shown in Fig. 3(b) where

$$Y_r' = Y_r n_r^2$$
$$Z_r = X_r + X_r'. \tag{17}$$

Applying this procedure to every series element and then retaining the series elements of admittance Y_r', we have the typical coupling network shown in Fig. 3(c) with a transfer matrix

$$\begin{bmatrix} \frac{1}{n_r} & 0 \\ 0 & n_r \end{bmatrix}\begin{bmatrix} 1 & jX_r' \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & jK_{r,r+1} \\ \frac{j}{K_{r,r+1}} & 0 \end{bmatrix}\begin{bmatrix} 1 & jX_{r+1} \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} n_{r+1} & 0 \\ & \frac{1}{n_{r+1}} \end{bmatrix}$$
$$= \begin{bmatrix} \frac{-X_r' n_{r+1}}{K_{r,r+1} n_r} & j\left[\frac{K_{r,r+1}^2 - X_r' X_{r+1}}{K_{r,r+1} n_r n_{r+1}}\right] \\ j\frac{n_r n_{r+1}}{K_{r,r+1}} & \frac{-X_{r+1} n_r}{K_{r,r+1} n_{r+1}} \end{bmatrix} \tag{18}$$

which may be equated to the ideal phase shifter with a transfer matrix

$$\begin{bmatrix} \cos\psi_{r,r+1} & j\sin\psi_{r,r+1} \\ j\sin\psi_{r,r+1} & \cos\psi_{r,r+1} \end{bmatrix} \tag{19}$$

to yield the set of recurrence formulas for $r = 1 \to (n-1)/2$ (n odd):

$$n_{r+1} = \frac{n_r K_{r,r+1}}{\sqrt{n_r^4 + X_r^2}}$$

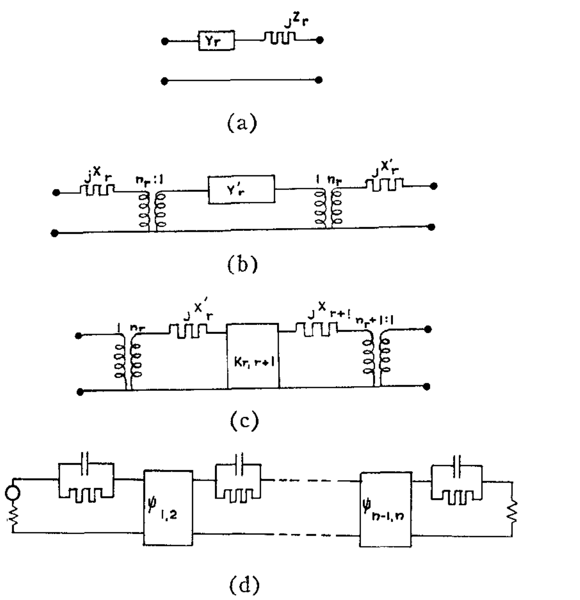

Fig. 3. (a) Typical series element. (b) Impedance transformation for series element. (c) Typical coupling element. (d) Transformed prototype network for bandstop filter.

$$X_{r+1} = Z_{r+1} - X_{r+1}' = X_r'\left(\frac{n_{r+1}}{n_r}\right)^2 \tag{20}$$

with

$$\psi_{r,r+1} = -\tan^{-1}\left[\frac{n_r^2}{X_r'}\right]. \tag{21}$$

The initial conditions are

$$n_1 = 1, \qquad X_1' = Z_1 \tag{22}$$

and are chosen to retain the normalized generator impedance of unity.

Since the basic prototype inherently possesses complex conjugate symmetry, it is only necessary to evaluate the recurrence formulas up to the central element since the transformed network will also possess complex conjugate symmetry. Furthermore, since n is odd, at $\omega=\infty$ there is perfect transmission ensuring the cancellation of the frequency invariant reactances at the center. Consequently, the final network is as shown in Fig. 3(d).

To convert this prototype into a waveguide bandstop filter, we apply the familiar frequency transmission

$$\omega \to \alpha\left(1 - \frac{\lambda_g}{\lambda_{g0}}\right) \tag{23}$$

where

α $=(\lambda_{g1}+\lambda_{g2})/(\lambda_{g1}-\lambda_{g2})$;
λ_{g1} guide wavelength at lower stopband frequency;
λ_{g2} guide wavelength at upper stopband frequency;
λ_{g0} $=(\lambda_{g1}+\lambda_{g2})/2$, guide wavelength at center stopband frequency.

The unity impedance phase shifters are simply realized by an equivalent length of uniform waveguide at $\lambda_g=\lambda_{g0}$ and may be realized to within any half-wavelength integer. In practice, the lengths of these coupling guides should be between one-half and one wavelength long to prevent evanescent mode interaction and avoid excessive length.

The resonant stubs are realized by inductive aperture-coupled series stubs of uniform guide with an admittance, normalized to the main guide of

$$-jB_r'\frac{\lambda_g}{\lambda_{g0}} - j\cot\left[\phi_r\frac{\lambda_{g0}}{\lambda_g}\right] \tag{24}$$

where B_r' is the midband susceptance of the iris and ϕ_r the electrical length of the cavity, which is normally in the range $(\pi/2)<\phi_r<\pi$.

From (16), (17), and (23), the required susceptance for the rth resonant stub is

$$n_r^2\left[\alpha C_r\left(1 - \frac{\lambda_g}{\lambda_{g0}}\right) + B_r\right] \tag{25}$$

which is equated to (24) and its derivative at midband to give

$$B_r' + \cot\phi_r = -B_r n_r^2$$
$$B_r' + \phi_r(1+\cot^2\phi_r) = \alpha C_r n_r^2 \tag{26}$$

resulting in ϕ_r being evaluated iteratively through the equation

$$\phi_r(1+\cot^2\phi_r) - \cot\phi_r = (\alpha C_r + B_r)n_r^2 \tag{27a}$$

or

$$\frac{2\phi_r - \sin 2\phi_r}{1-\cos 2\phi_r} = (\alpha C_r + B_r)n_r^2 \tag{27b}$$

with the initial approximate value of

$$\frac{\phi_r}{\pi} = 1 - \frac{1}{n_r}\sqrt{\frac{1}{\pi(\alpha C_r + B_r)}}. \tag{28}$$

Then B_r' is obtained from

$$B_r' = -[n_r^2 B_r + \cot\phi_r]. \tag{29}$$

In summary, for the symmetrical bandstop response shown in Fig. 4(a) the element values for the filter shown in Fig. 4(b) are obtained as follows.

Given λ_{g1}, λ_{g2}, λ_{g1}', λ_{g2}', L_R, and L_S where

$$2\lambda_{g0} = \lambda_{g1} + \lambda_{g2} = \lambda_{g1}' + \lambda_{g2}'$$

compute

$$m = \left[\frac{\lambda_{g1} - \lambda_{g2}}{\lambda_{g1}' - \lambda_{g2}'}\right]^2 \tag{30}$$

and

$$\alpha = \frac{\lambda_{g1} + \lambda_{g2}}{\lambda_{g1} - \lambda_{g2}}.$$

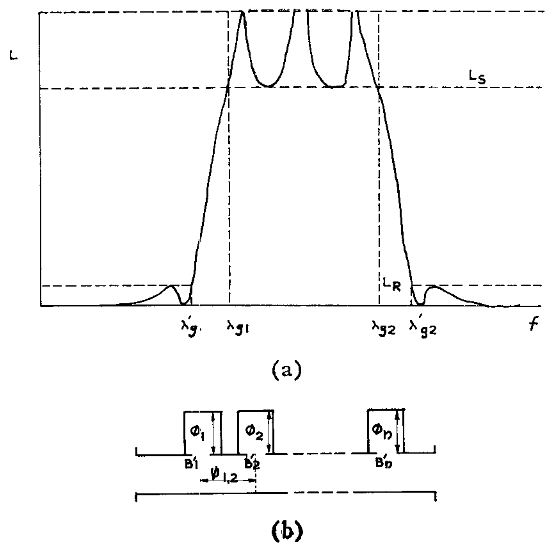

Fig. 4. (a) Bandstop insertion loss function. (b) Final configuration for bandstop filter.

Determine n from

$$L_R + L_S = 13.65 n \frac{K'(m)}{K(m)} - 12$$

and m_0 from

$$n \frac{K(m_0)}{K'(m_0)} = \frac{K(m)}{K'(m)} \cdot$$

Compute

$$\frac{1}{\epsilon} = 10 \uparrow [L_S/20]$$

$$U = \frac{K'(m)}{K'(m_0)} \, \mathrm{sc}^{-1} \left[\frac{1}{\epsilon} \,\Big|\, 1 - m_0 \right]$$

$$y = \mathrm{sc}\,[U \mid 1 - m]. \tag{31}$$

For $r = 1 \to (n+1)/2$, C_r, B_r, R_r, and $K_{r,r+1}$ are defined in (2)–(5) and n_r and $\psi_{r,r+1}$ are obtained from successive application of the recurrence formulas (20)–(22) for $r = 1 \to (n-1)/2$.

For $r = 1 \to (n+1)/2$, (27)–(29) are applied and finally, from the complex conjugate symmetry property,

$$\psi_{n-r,n-r+1} = 3\pi - \psi_{r,r+1}$$

$$\phi_{n-r+1} = \phi_r$$

and

$$B_{n-r+1}' = B_r' + 2n_r^2 B_r. \tag{32}$$

From a physical viewpoint it may be noted that the resonant frequencies of the stubs occur in consecutive order along the filter; i.e., if the transmission zeros occur at $f_1 \to f_n$ in increasing order, then these are realized by stubs $1 \to n$ along the filter. This configuration may be contrasted with the results obtained from the conventional prototype where transmission zeros necessarily occur in pairs that produce zeros with geometrical symmetry about the band center.

For the specific case of the inverse Chebyshev response ($m = 0$), the design equations simplify considerably to the direct design equations:

$$\alpha = \frac{\lambda_{g1} + \lambda_{g2}}{\lambda_{g1} - \lambda_{g2}}$$

$$\frac{1}{\epsilon} = 10 \uparrow [L_S/20] \qquad y = \sinh n \sinh^{-1} \frac{1}{\epsilon} \tag{33}$$

$$\frac{2\phi_r - \sin 2\phi_r}{1 - \cos 2\phi_r} = \frac{1}{2y \sin \left[\frac{(2r-1)\pi}{2n} \right]} \cdot \left[\alpha + \cos \left[\frac{(2r-1)\pi}{2n} \right] \right] \tag{34}$$

$$B_r' = \left[\frac{1}{2y \tan \left[\frac{(2r-1)\pi}{2n} \right]} + \cot \phi_r \right] \tag{35}$$

$$\psi_{r,r+1} = \frac{3\pi}{2} \cdot \tag{36}$$

Conclusions

A direct design procedure has been presented for waveguide elliptic function filters from explicit formulas in the natural prototype. The results have been restricted to the odd-degree case to enable a uniform guide to be used. For the even-degree case, the introduction of a small discontinuity in the center of the filter will allow the same design procedure to be used.

The final form of network is the familiar uniform guide with series stubs that have previously been used for the realization of convention maximally flat and Chebyshev filters [5]. This extension to the inverse Chebyshev and elliptic function responses will enable more stringent specifications to be met by the same physical device.

References

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